

# Low $x$ Physics

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In this talk, we present the arguments, that a new QCD regime - gluon saturation, has been reached at HERA.

## I. MAIN QUESTIONS AND PROBLEMS

We hope that everybody agrees, we have two principle problems in low  $x$  physics: (i) matching between “soft” (npQCD) and “hard” (pQCD) processes; and (ii) theoretical description of high parton density QCD (hdQCD). My credo is [1–3]: *these two problems are correlated and the system of partons always passes the stage of hdQCD ( at shorter distances ) before it goes to the black box, which we call non-perturbative QCD and which, in practice, we describe in old fashion Reggeon phenomenology.*

In this talk you can find the answer to the following questions:

- Is there any violation of the DGLAP evolution?
- Can we describe matching between “soft” and “hard” processes?
- Do we see a signal of the gluon saturation in HERA data?
- What is a current situation in theory for high parton density QCD system?

Unfortunately, the lack of space does not allow us to discuss such a hot problem as the status of the BFKL Pomeron as well as the manifestation of high parton density QCD in the Tevatron data.

## II. TWO SCALES OF DIS

About thirty years ago Gribov [4] noticed that photon - hadron interaction at high energies has two distinct stages as far as time - space picture of interaction is concerned:

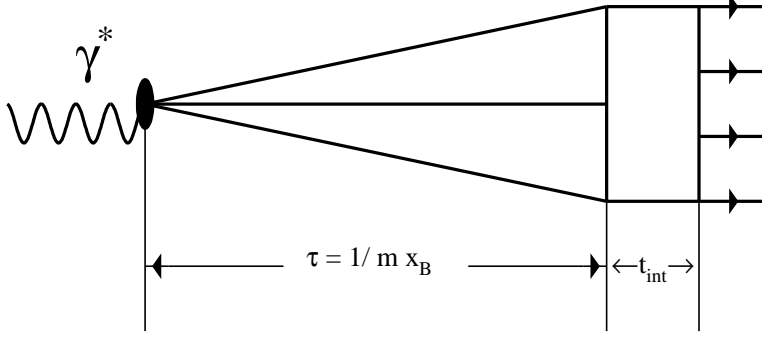


FIG. 1.

1.  $\gamma^* \longrightarrow$  hadron system (  $q\bar{q}$  - pair );
2. hadron system (  $q\bar{q}$  - pair ) interacts with the target.

Therefore, we can describe photon-hadron interaction as follows

$$\sigma_{tot}(\gamma^* p) = \sum_n |\Psi_n|^2 \sigma_{tot}(np) , \quad (1)$$

where  $n$  denotes the set of quantum numbers which diagonalize the high energy interaction matrix. This set of quantum numbers we call the correct degrees of freedom (DOF) and  $\sigma_{tot}(np)$  is the total cross section for the interaction of the hadron or parton system with quantum numbers  $n$  with the target. Eq. (1) is useful only if we know the correct DOF. Fortunately, we do know them at short distances where  $n$  are colour dipoles [5] and Eq. (1) reads as

$$\sigma_{tot}(\gamma^* p) = \int d^2 r_t \int_0^1 dz |\Psi(Q^2; r_t, z)|^2 \sigma_{tot}(r_t^2, x) . \quad (2)$$

#### A. Separation scale $r_\perp^{\text{sep}} = 1/M_0$

However, at long distances we do not know the correct DOF. The scale which says what distances are short we call a separation scale (see Table.1).

**Table 1.**

$r_{\perp} <$	$r_{\perp}^{sep}$	$< r_{\perp}$
$\Psi(Q^2; r_t, z) \rightarrow$ pQCD		$\Psi(Q^2; r_t, z) \rightarrow$ npQCD
DOF: colour dipoles [5]		DOF: constituent quarks [6] colour dipoles [7]
$\sigma_{tot}(r_{\perp}, x) \propto$ $r_{\perp}^2 xG(x, 4/r_{\perp}^2)$		$\sigma_{tot}(r_{\perp}, W) \rightarrow$ Regge phenomenology

### B. Saturation scale $r_{\perp}^{sat} \approx 1/Q_s(x)$

This scale can be estimated from the equation

$$\kappa = \frac{3\pi^2\alpha_S}{2Q_s^2(x)} \times \frac{xG(x, Q_s^2(x))}{\pi R^2} = 1, \quad (3)$$

which says that the packing factor of partons in the parton cascade is about unity or, in other words, at this scale the parton system is so dense that we cannot apply the standard methods of perturbative QCD.

The physical meaning of  $\kappa$  is clear from Fig. 2 which shows the parton distributions in the transverse plane. At short distances  $\kappa$  is the product of the parton density in transverse plane which is equal to *number of partons/area*  $= xG(x, Q^2)/\pi R^2$  multiplied by the cross section of the parton interaction  $\propto \alpha_S/Q^2$ . Therefore, at the scale where  $\kappa \approx 1$  the interaction between partons become essential and new approach should be developed in QCD to describe the high parton density system.

The natural hierarchy of the scales is  $r_{\perp}^{sat} \ll r_{\perp}^{sep}$  accordingly to our main idea.

### III. THEORY STATUS

The theory of high density system in QCD is in a very good shape now. The problem has been attacked from two different point of view: (i) from the pQCD region [1,2,8] by summing corrections to the evolution equations due to high density of partons; and (ii) from the non-perturbative QCD region by developing effective Lagrangian approach [3,9] dealing with such a system. The resulting evolution equation that has been proven [10] looks as follows

$$\begin{aligned} \frac{da^{el}(\vec{x}_{01}, b_t, y)}{dy} &= -\frac{2C_F\alpha_S}{\pi} \ln\left(\frac{x_{01}^2}{\rho^2}\right) a^{el}(\vec{x}_{01}, b_t, y) + \frac{C_F\alpha_S}{\pi} \int_{\rho} d^2x_{12} \frac{x_{01}^2}{x_{02}^2 x_{12}^2} \\ &\cdot \left( 2 a^{el}(\vec{x}_{02}, \vec{b}_t - \frac{1}{2}\vec{x}_{12}, y) - a^{el}(\vec{x}_{02}, b_t, y) a^{el}(\vec{x}_{12}, b_t, y) \right), \end{aligned} \quad (4)$$

where  $a^{el}$  is the elastic amplitude for dipole scattering at fixed impact parameter  $b_t$  and at energy  $s$  ( $y = \ln s$ ).

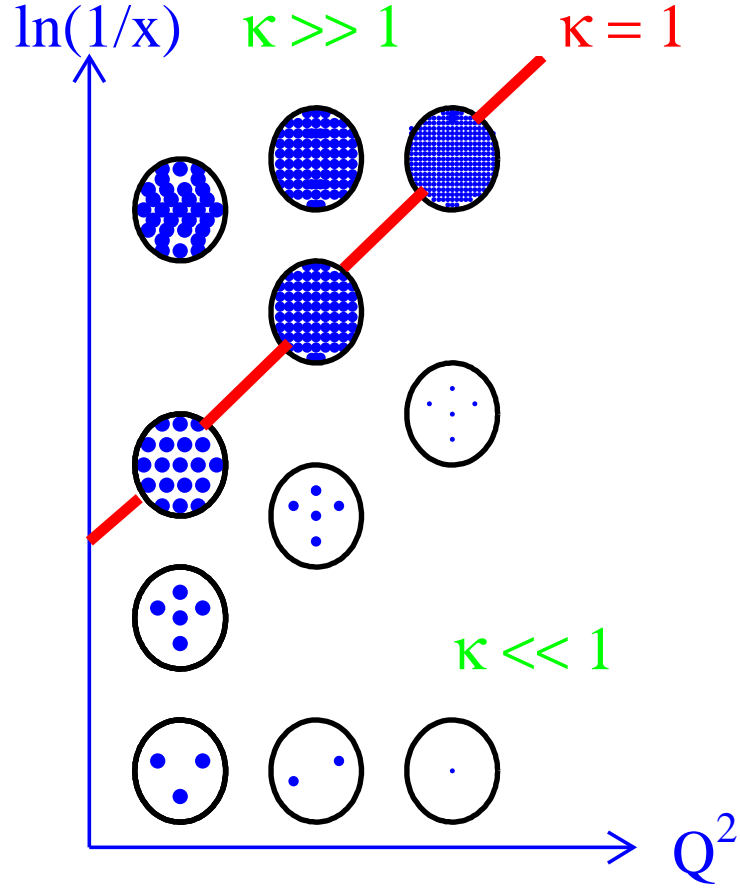


FIG. 2.

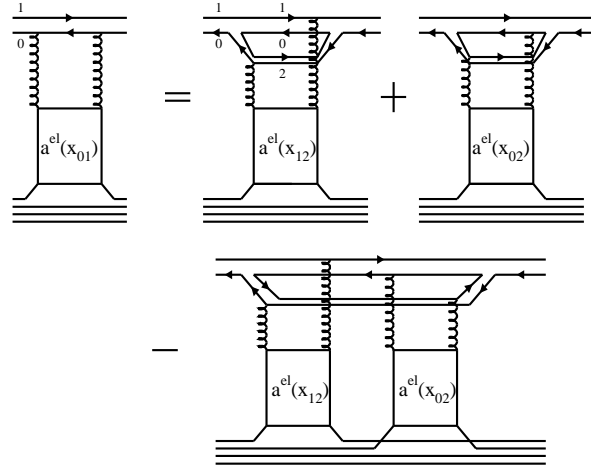


FIG. 3.

The physical meaning of Eq. (4) is very clear from Fig. 3. Indeed, with probability  $\frac{x_{01}^2}{x_{02}^2 x_{12}^2}$  dipole with size  $x_{01}^2$  decays in two dipoles with sizes  $x_{02}^2$  and  $x_{12}^2$  (see Fig. 3). These two dipoles interact with the target: each produced dipole interacts with the target separately (the second term in Eq. (4)) or two

dipoles interact with the target simultaneously ( the third term in Eq. (4) ). The first term describes the fact that dipole  $\mathbf{x}_{01}^2$  disappears from the initial state after decay into two dipoles. This equation, which has been suggested in the momentum representation in Ref. [1], has a lot of nice properties including the correct matching with the DGLAP evolution equations. However, the most important message concerning this equation is that this equation can be derived using the effective Lagrangian approach [3]. It should be stressed that we know not only the equation but also the initial condition for it.

#### IV. MATCHING OF “SOFT” AND “HARD” PHOTON-PROTON INTERACTIONS

Serious attempts [11] to find the value of the separation scale  $r_{\perp}^{sep} = 1/M_0$  were undertaken using Gribov’s formula [4] during the past decade, starting from pioneering paper of Kwiecinski and Badelek [12]. Gribov’s formula reads (see Fig. 5)

$$\sigma(\gamma^* N) = \frac{\alpha_{em}}{3\pi} \int \frac{\Gamma(M^2)dM^2}{(Q^2 + M^2)} \sigma(M^2, M'^2, s) \frac{\Gamma(M'^2)dM'^2}{(Q^2 + M'^2)} \quad (5)$$

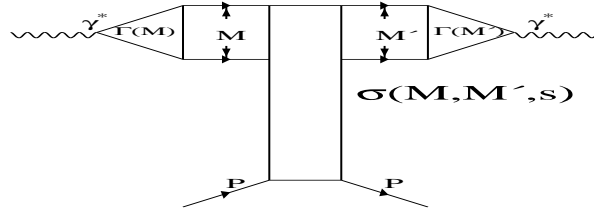


FIG. 4.

The following assumptions are made to describe the matching between “soft” and “hard” processes:

$$\begin{array}{ll} M < M_0 & M > M_0 \\ M = M' & M \neq M' \end{array}$$

D-L Pomeron + AQM      “Hard” Pomeron  $\equiv$  pQCD approach

The result ( see Fig. 5 ) is that the separation scale depends on polarization of the incoming photon and it is equal  $0.7 < M_{0T}^2 < 0.9 \text{ GeV}^2$  for transverse polarized photon and it is smaller for the longitudinal polarized photon (  $M_{0L}^2 < 0.4 \text{ GeV}^2$  ).

#### V. WHERE ARE SC?

The HERA data put a puzzling question on the table:

- On one hand, the data can be described by the routine DGLAP evolution equations [13];
- On the other hand, the gluon density measured at HERA is so high that some effect of hdQCD should be seen (see Fig. 6 where parameter  $\kappa$  is plotted as it appears in HERA data. )

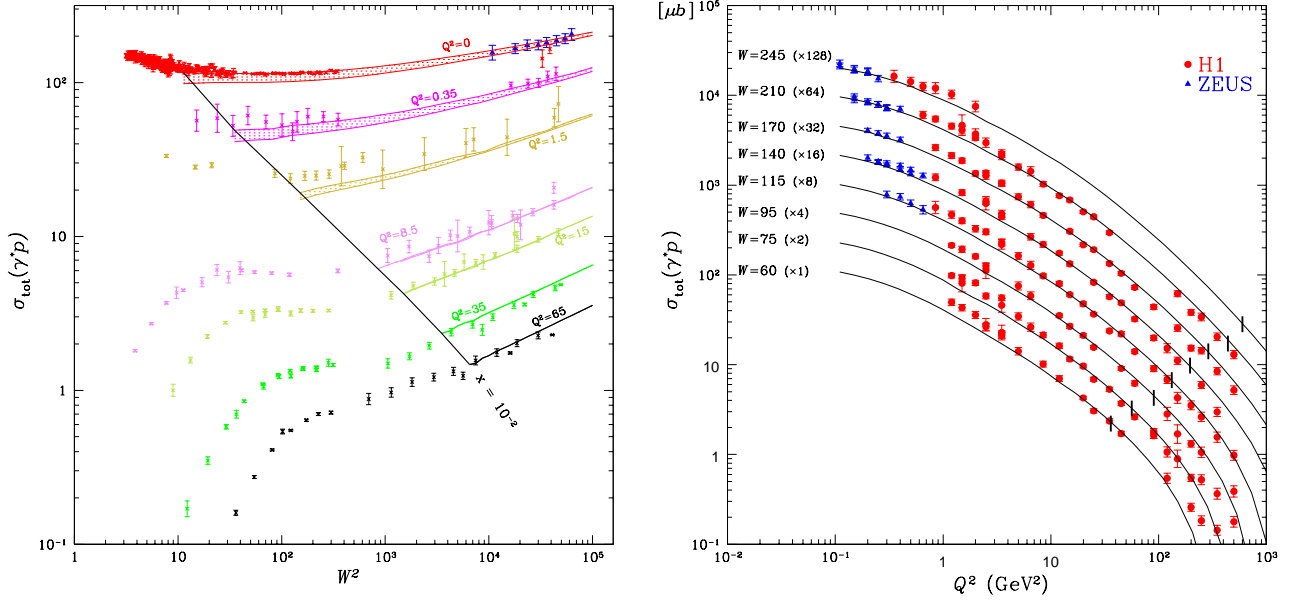


FIG. 5. An example of description the HERA experimental data using Gribov's formula, taken from GLMN paper [11]

The natural question to ask is where to look for a saturation scale  $Q_s(x)$ . Fig. 7 shows us that the high parton density corrections to  $F_2$  is rather small while they are substantial for the gluon structure function.

## VI. $Q_s^2(x)$ FROM $Q^2$ - DEPENDENCE OF $F_2$ - SLOPE

There is a hope that the  $F_2$ -slope ( $\frac{\partial F_2}{\partial \ln Q^2}$ ) will provide a measurement of the saturation scale since in the DGLAP evolution this slope  $\frac{\partial F_2}{\partial \ln Q^2} = \frac{2\alpha_s}{9\pi} x G^{DGLAP}(x, Q^2)$  directly proportional to the gluon structure function. On the other hand, a gluon saturation leads to the slope which is proportional [14] to  $Q^2 R^2$  at fixed  $x$  where  $R$  is the target size. Indeed, it turns out that hdQCD corrections [14] are able to describe all experimental data on the  $F_2$ -slope (see Fig.8). We consider as an important sign, that we are on the right track, the fact that two DGLAP parameterizations GRV'94 and GRV'98 lead to a good description of the experimental data after taking into account hdQCD effects.

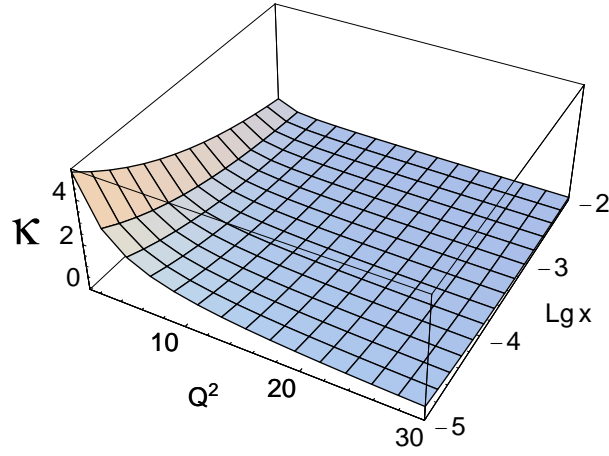


FIG. 6.

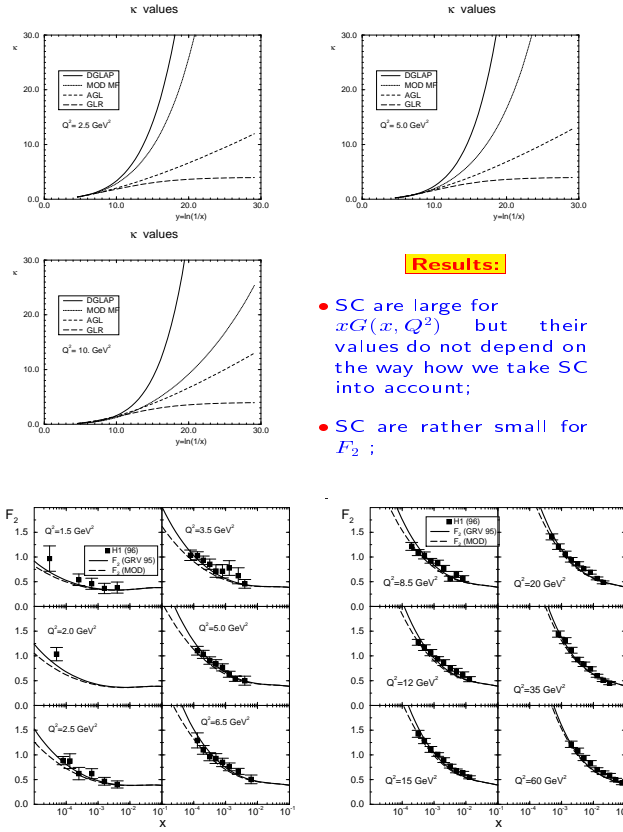


FIG. 7. The hdQCD corrections to  $F_2$  and  $xG(x, Q^2)$  calculated in Ref. [15]. Calculation were done using Eq. (4).

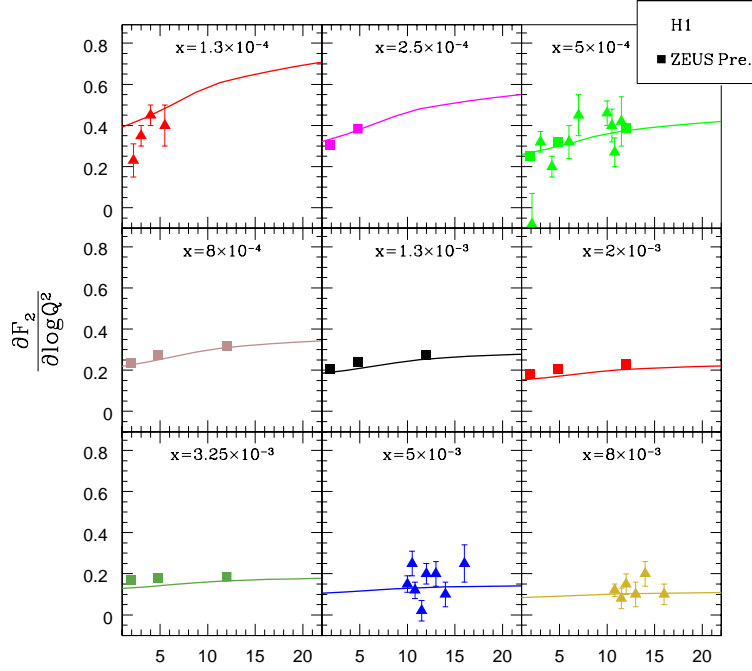


FIG. 8.

However, the  $F_2$ -slope data can be equally well described by “soft” and “hard” contribution without hdQCD effects if we assume that the “soft” contribution stems from rather short distance  $0.3 \div 0.5 \text{ fm}$  [7] (see Fig.9).

## VII. HIGH DENSITY QCD AND DIFFRACTIVE $J/\Psi$ -PRODUCTION IN DIS

It turns out that it is very instructive to consider two observables : the  $F_2$ - slope and the  $J/\Psi$ -production in DIS. The hdQCD effect are essential in both observables and a simultaneous analysis of them could give an information on the value of the saturation scale. Performing such an analysis we obtain the following: (i) none of MRS parameterizations survives; (ii) all GRV parameterizations survive only with hdQCD effects; and (ii)  $\chi^2/\text{n.d.f.}$  is excellent for GRV + hdQCD effects (see Fig. 10 ).



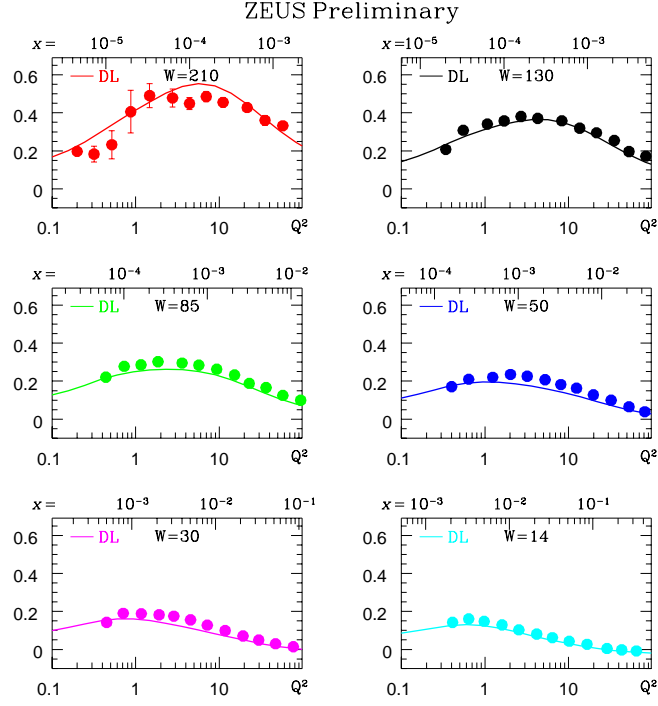


FIG. 9. The  $F_2$ -slope in D-L model with “soft” contribution at rather short distances [16]

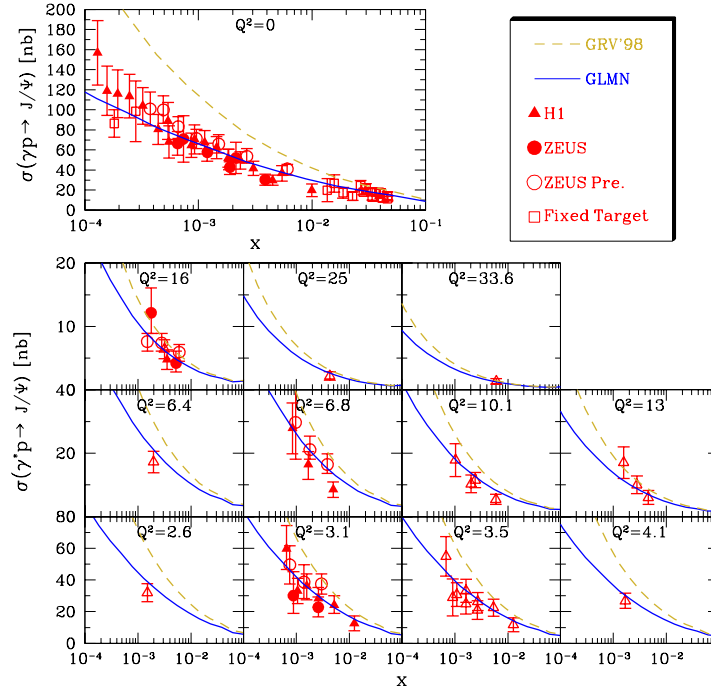


FIG. 10.  $J/\psi$  production in hdQCD approach. Curves are taken from Ref. [17]

## VIII. A NEW SCALING

The saturation hypothesis got a new support by Golec-Biernat and Wüsthoff [18] who suggested a simple model that introduces only one scale for colour dipole - target interaction: the saturation scale  $Q_s(x)$ , which they defined by fitting experimental data. This simple model does not take into account even such well established property as scaling violation but describes all experimental data from HERA for  $x < 0.01$ .

In Ref. [19] it was noticed that the HERA data show a new scaling:  $\sigma(\gamma^*p) = R^2 F(Q^2/Q_s^2(x))$  for all values of  $Q^2$  at  $x < 0.01$  ( see Fig. 11 ). Such a scaling was expected [1,20,3,21] in the saturation region to the left of the critical line  $\kappa = 1$  in Fig.2. The value of the saturation scale  $Q_s(x)$  can be measured as a value of  $Q^2$  at which we see a deviation from this scaling. The fact, that all data at HERA kinematic region for  $x < 0.01$  show this scaling, confirms the idea that at  $x < 0.01$  DIS is deeply in the saturation region. It should be stressed that the whole idea of saturation is the simple fact that the only one scale  $r_{saturation}$  determines the scattering amplitude for the distances longer than  $r_{saturation}$ . This very fact one can see directly in Fig.2, noticing that a hadron looks as the diffraction grid with the size  $r_{saturation}$  in the saturation region ( $\kappa \gg 1$  in Fig. 2 ).

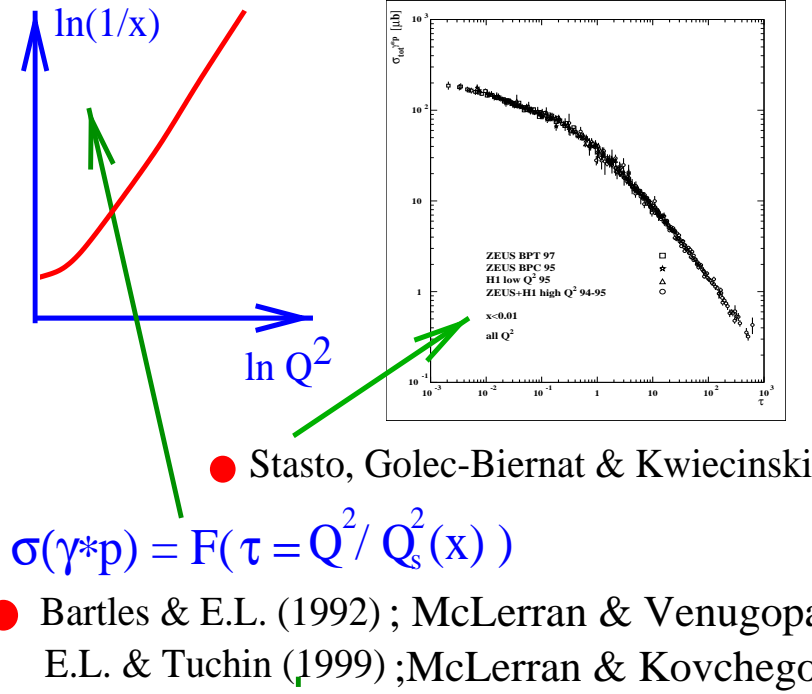


FIG. 11.

## IX. SUMMARY

The brief review on low  $x$  physics, which is given in this talk, allows us to make a definite conclusion, that a new QCD regime has been reached at HERA: the regime of high parton density QCD with a gluon saturation. Let us list all arguments for such a new regime:

- There are no experimental data that are in a contradiction with the asymptotic prediction of high density QCD. Indeed,
  - The  $F_2$ -slope shows  $dF_2/d\ln Q^2 \propto Q^2$  behaviour at  $Q^2 < Q_s^2(x)$  [22,23];
  - The ratio of diffraction cross section in DIS to the total DIS cross section is constant at HERA kinematic region [21];

- The inclusive diffraction stems from short distances as should be in a gluon saturation picture in which a hadron looks as a diffraction grid with a typical size  $1/Q_s(x)$  [24,25];
- The HERA data show a new scaling [19], predicted theoretically [1,20,3,21] in the saturation region, namely, that  $\sigma(\gamma^*p) = R^2 F(Q^2/Q_s^2(x))$  for all values of  $Q^2$  at  $x < 0.01$ .
- All data can be described in the simplest saturation model of Golec-Biernat and Wüsthoff [18];
- The current parameterizations, based on the DGLAP evolution equations, cannot describe simultaneously the  $Q^2$ -behaviour of the  $F_2$  slope and energy behaviour of the  $J/\Psi$  production in DIS and photoproduction [17];
- The theoretical developments in high density QCD has been so remarkable during the past two years, that we can trust Eq. (4) which predicts the essential high density collective phenomena in HERA kinematic region;
- The simple estimates based on the gluon density extracted from the HERA data show large packing factor (see Fig. 6) or, in other words, they show the strong shadowing corrections are needed in HERA kinematic region.

In spite of everything mentioned above, not everybody will agree with the strong statement that has been formulated here. The reason for this is very simple: most of data, that has been considered above, have different explanations without SC. All these alternative explanations cannot be considered as natural ones, but it is behind many people scepticism on a new QCD regime at HERA, that the DGLAP evolution has more fundamental origin in QCD than all estimates with SC. It is not true at all and SC are more general approach than the DGLAP equation because (i) they are consistent with the  $s$ -channel unitarity; and (ii) the non-linear equation (see Eq. (4)) has the same deep operator proof as the DGLAP evolution equation.

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